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## GEOMETRICAL MODELING OF DEFORMATION DIAGRAMS AND STRENGTH CRITERION OF COMPOSITE STEEL-CONCRETE RODS

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*Key words:* steel-concrete rods, stress-strain state, deformation diagrams, strength criterion, destruction.

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*This article presents a mathematical model and a geometric criterion for determining the load-bearing capacity of concrete-filled steel tube bars. A function for deforming composite rods is proposed, which makes it possible to determine the values of critical loads. The results obtained can be used in application packages for computer-aided design.*

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### Introduction

Steel-reinforced concrete structures have been widely used in construction in recent years, and therefore they are gaining increased interest from both domestic [1–3] and foreign [4–6] scientists. Experimental, numerical, and analytical studies are actively conducted. However, the existing regulatory documentation still has a sufficient number of unlit and controversial issues, and calculation methods do not accurately describe the stress-strain state of steel-concrete rods.

This study presents a mathematical model obtained as a result of processing experimental studies of short steel-concrete rods subjected to an axial compressive load. In addition, a criterion for the loss of bearing capacity is proposed, at which the deformations of the rod exceed the permissible ones.

### Materials and methods

To construct mathematical models of deformation used in the development of a geometric criterion for the loss of bearing capacity of small-section steel-concrete rods, laboratory samples were prepared consisting of straight-seam electric-welded steel tubes filled with a concrete core. Round tubes with sections  $60 \times 2.0$ ,  $76 \times 3.0$  и  $102 \times 3.5$  (with a diameter ratio to the wall thickness  $D/t_w = 25 \dots 30$ ). As a material for the concrete core, a highly mobile concrete mixture was used, prepared from a cement-sand mixture of grade M300, fine-grained gravel and water without impurities and additives. Additional reinforcement of the core with flexible rod fittings was not used due to the tightness of the cross sections. The strength characteristics of the resulting concrete were determined by the fact of strength gain by testing control concrete specimens at the age of 28 days by the destructive method.

To evenly transfer the load from the loading plates of the press to the entire area of the composite tube-concrete section, the ends of the steel tubes were milled, filled with a highly mobile concrete mixture with an excess and subsequent facing using a diamond

disc. In addition to steel-concrete rods, hollow specimens of steel tubes were also made for each standard size.

To test specimens with a length of 100 mm, an experimental installation was used (Fig. 1) equipped with a P-125 press with an electrohydraulic drive, with which an axial compressive load was applied to tube concrete specimen installed between the loading plates. The loading plates were brought into a hinged state, which ensures the rotation of the plates during loading. The load was applied smoothly and is considered short-term, and therefore the creep phenomenon is not taken into account. The registration of the load created on the specimen was carried out using an analog dial of the press force meter.

During the entire period of loading the specimens, using the clock type indicator S379 and IH 50×0.01, the convergence of the press plates was recorded with an accuracy of 0.01 mm, corresponding to the complete longitudinal deformation of the rod during compression.

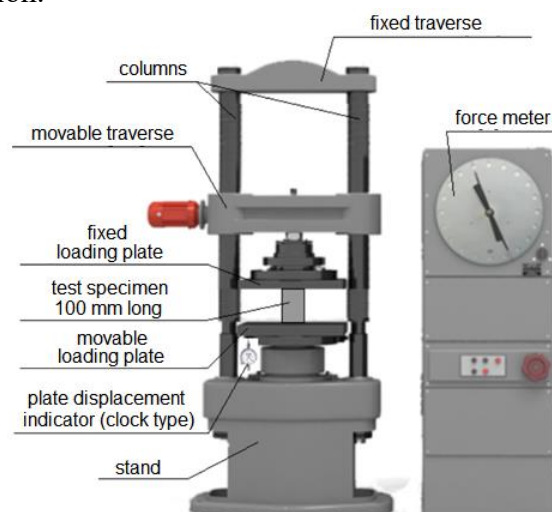


Fig. 1. Scheme of the experimental setup for testing steel and steel-concrete specimens with axial compressive load in P-125 test press [7]

The results of the experimental studies were processed and presented in the form of longitudinal deformation diagrams (Fig. 2), the destructive load was recorded. In addition to the studied diagrams for steel concrete specimens, graphs for hollow steel tube were also constructed.

The loss of the bearing capacity of the hollow tube in all cases occurred due to local loss of wall stability with the formation of a transverse fold (Fig. 3a). Short steel-concrete specimens made of tubes of appropriate geometric parameters with concrete filling withstood a load significantly exceeding the bearing capacity of the tube, since the concrete core prevents premature loss of wall stability (Fig. 3b). The results of previous studies [7] show that the total bearing capacity of the composite section significantly exceeds the sum of the differentiated bearing capacities of the tube and the concrete core cylinder. This is due to the complex stress-strain state of triaxial compression, in which concrete continues to work together with a steel tube, despite the deformations beyond the limits for concrete. A detailed study of the destroyed specimens during their opening showed that the concrete was deformed absolutely plastically, without cracking (Fig. 3b).

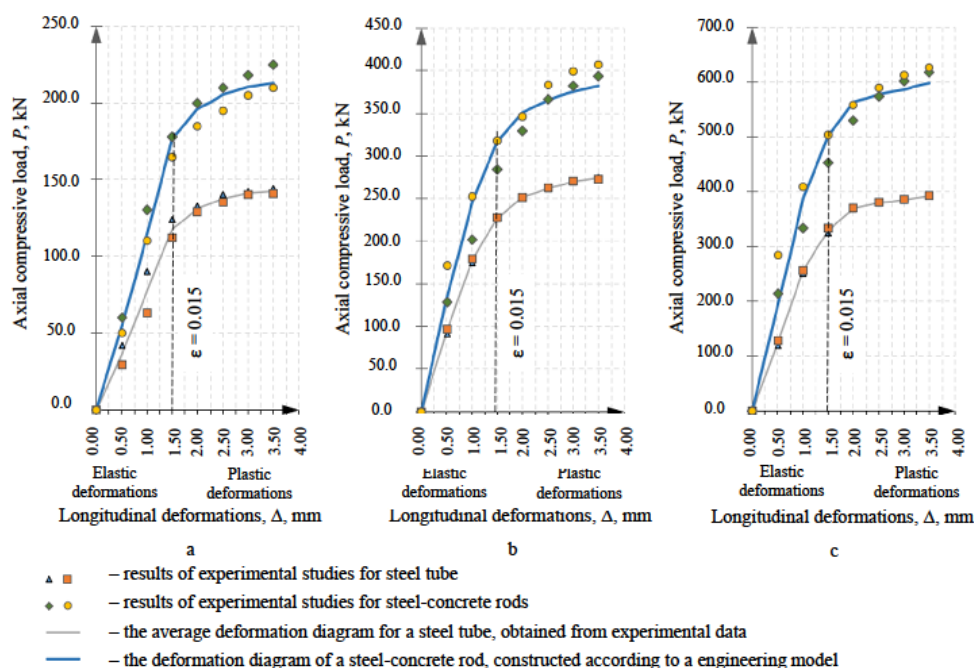


Fig. 2. Diagrams of longitudinal deformation of steel and steel-concrete specimens under axial compression, obtained experimentally and using an engineering model: tube 60×2.0 (a), tube 76×3.0 (b), tube 102×3.5 (c)



Fig. 3. Steel tube (a) and tube specimens (b) specimen after axial compressive load tests; deformed state of the concrete core after the transition to the plastic phase (c)

There is an obvious relationship between the graphs (Fig. 2) – the diagram for rods with a concrete core is similar to the diagram for a hollow tube with the appropriate dimensions. In addition, the transition from the elastic stage of work to the plastic stage for both tube and tube concrete occurs at the same deformations (approximately equal to  $\epsilon = 0.015$ ). Thus, the presence of a core in the tube makes a certain contribution to the bearing capacity of the structure, and this contribution is constant at each stage of deformation, which causes the similarity of the studied diagrams. The method proposed

in [8] is also based on the assumption that the contribution of concrete is commensurate at all stages of loading.

In the article [9], an engineering model of deformation of short tubular concrete rods is proposed, with the help of which the value of the axial compressive force  $P$  can be calculated for an arbitrary value of longitudinal deformation  $\Delta$ :

$$P_{tb}(\Delta) = P_t(\Delta) \cdot \left(1 + 2,5 \cdot \frac{P_b^{cr}}{P_t^{cr}}\right), \quad (1)$$

where  $P_{tb}(\Delta)$  and  $P_t(\Delta)$  – the longitudinal force in the steel-concrete rods and steel tube, corresponding to the longitudinal movement of the ends  $\Delta$ ;

$P_t^{cr}$  и  $P_b^{cr}$  – the maximum bearing capacity of a steel tube (cage) and a concrete core during their separate operation, which can be determined according to existing regulatory methods for design.

The proposed dependence makes it possible to estimate the order of deformations and the rate of deformation of compressed steel concrete rods of low flexibility and is suitable for calculating elements with different cross-section sizes. However, this technique is not universal, which could analytically describe the deformation of tube concrete for subsequent implementation into calculation complexes, since in order to obtain the desired graph  $P_{tb}(\Delta)$ , it is necessary to have a graph for a steel pipe  $P_t(\Delta)$ .

#### The results of the researches

During the analysis of the obtained diagrams, analytical dependencies were selected that allow describing the deformation process of steel-concrete elements using mathematical modeling methods. In this case, the diagram is presented in the form of a continuous piecewise given function (Fig. 4): linear  $AB$  in the area with relative deformations  $0 \leq \varepsilon \leq 0,01$  (elastic deformation area) and logarithmic  $BC$  at large deformations ( $\varepsilon > 0,01$ ):

$$\begin{cases} P_{tb}(\Delta) = A \cdot \lg(k_\Delta) \cdot \Delta & \text{при } 0 \leq \varepsilon \leq 0,01 \\ P_{tb}(\Delta) = A \cdot \lg(k_\Delta \cdot \Delta) & \text{при } \varepsilon > 0,01 \end{cases}, \quad (2)$$

where  $\Delta$  – axial shortening of the sample (offset of the ends), mm;

$A$  – a constant value for the studied composite structure, depending on the geometric characteristics of the sections, stiffness and strength of the materials used,

$$A = k_m \cdot P_t^{cr} \cdot \left(1 + k_b \cdot \frac{P_b^{cr}}{P_t^{cr}}\right), \quad (3)$$

$k_m = 0,85$  – the scaling factor of the model;

$k_b = 2,5$  – a coefficient that takes into account the contribution of concrete to the work of the structure;

$k_\Delta = 5$  – the scaling factor of the movements.

The logarithmic function  $BC$  is selected so that it corresponds to the actual deformation process obtained by constructing and processing diagrams based on experimental data. The linear function (line  $AB$ ) is determined from the condition that the line passes through the origin (point 0; 0) and the condition of continuity of the deformation model – the linear function must continuously transform into a logarithmic one (common point  $B$ ) [10].

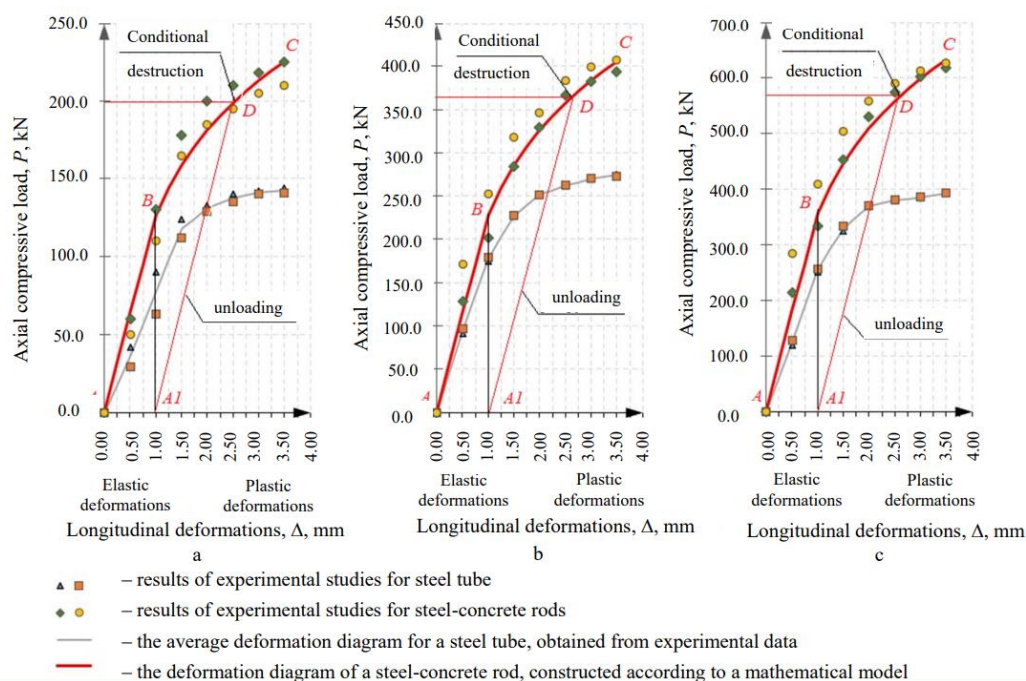


Fig. 4. Diagrams of longitudinal deformation of steel and steel-concrete specimen under axial load compression, obtained experimentally and using a mathematical model in the form of a piecewise set function: pipe 60×2.0 (a), pipe 76×3.0 (b), pipe 102×3.5 (c)

The deformation diagrams [7] show that during laboratory tests with a compressive load, deformations increase indefinitely without causing obvious destruction of the sample. There is a need to introduce an unambiguous criterion for the loss of bearing capacity, i.e. compressive force, at which further operation of the structure is impossible.

Destruction is a process that develops over time and can occur at different stress levels. In this regard, when designing structures made of elastically deformable materials, the calculation is made according to the yield strength, which certainly guarantees the strength of the structural element. For materials whose deformation diagrams do not have a pronounced yield point, the stress value  $\sigma_{cond}$  is taken as the conditional yield strength, at which the residual relative deformation of the specimen reaches a certain critical value  $\varepsilon_{res}^{cr}$ .

Since the composite steel-concrete rod has an inhomogeneous cross-section consisting of materials with different strength and deformation characteristics, it is not the classical deformation diagram in stresses and relative deformations  $\sigma - \varepsilon$  that is used, but the dependence between the longitudinal compressive force and the absolute shortening of the test sample  $P - \Delta$ .

In the graph of a piecewise defined function, point B is the boundary value between the linear and logarithmic sections. It is proposed to take the longitudinal force corresponding to the intersection of the logarithmic curve BC and the «unloading line» A1D, which is parallel to the initial stage of deformation AB and corresponds to the critical value of residual deformations, for the moment of conditional destruction of the steel-concrete rod  $\Delta_{res}^{cr}$ .



$$\begin{cases} P_{tb}(\Delta) = lg(k_{\Delta}) \cdot (\Delta - \Delta_{res}^{cr}) \\ P_{tb}(\Delta) = lg(k_{\Delta} \cdot \Delta) \end{cases}, \quad (4)$$

Equating the right-hand sides of the above functions, we obtain an equation that is a criterion for the loss of bearing capacity of a tubular concrete rod:

$$lg(k_{\Delta}) \cdot (\Delta^{cr} - \Delta_{res}^{cr}) = lg(k_{\Delta} \cdot \Delta^{cr}), \quad (5)$$

where  $\Delta_{res}^{cr} = \varepsilon_{res}^{cr} \cdot L_0$  – the critical absolute value of the residual axial deformations of the specimen during unloading, mm;

$L_0$  – the initial length of the specimen, mm.

The above equation has no analytical solution, however, the graphs of the studied functions (Fig. 4) indicate that the equation uniquely has a single solution that can be obtained by numerical or graphoanalytic methods. The solution of the equation is the critical deformation  $\Delta^{cr}$ , by which the critical force corresponding to the conditional loss of bearing capacity can be determined  $P_{tb}^{cr}$ , according to the formula:

$$P_{tb}^{cr} = A \cdot lg(k_{\Delta} \cdot \Delta^{cr}), \quad (6)$$

The numerical solution of equation (5) for the investigated standard sizes of pipe concrete specimens is given in Table.

Table

**Values of critical axial deformations and the conditional limit of the bearing capacity of steel-concrete specimens obtained by numerical solution**

The standard size of the tube	$\Delta^{cr}$ , mm	$\varepsilon^{cr}$	$P_{tb}^{cr}$ , kN	$P_t^{cr}$ , kN	$P_b^{cr}$ , kN	$\frac{P_{tb}^{cr}}{P_t^{cr} + P_b^{cr}} \cdot 100\%$
60×2.0	2.59	0,0259	201.3	142	28.2	118%
76×3.0	2.59	0,0259	362.4	275	43.5	114%
102×3.5	2.59	0,0259	564.9	392	82.3	119%

## Conclusion and discussion

Based on the results of the experimental studies and their analytical processing, the following conclusions were made:

1. The deformation of steel concrete rods of low flexibility occurs similarly to the deformation of a steel pipe, while the contribution of the concrete core to the overall bearing capacity is constant throughout the loading process. For both hollow and core specimens, the elastic deformation region is within the same limits.

2. A piecewise defined function is proposed that allows describing the deformation process by mathematical modeling methods. At small deformations at  $0 \leq \varepsilon \leq 0.01$ , the specimens are deformed elastically according to a linear law. At  $\varepsilon > 0.01$ , plastic deformations occur, the deformation rate increases, which corresponds to the logarithmic dependence of  $P - \Delta$ .

3. A geometric criterion for the loss of bearing capacity with a monotonously increasing deformation curve has been developed, which makes it possible to determine the value of the critical longitudinal compressive force at which the residual deformations exceed the permissible values. The proposed criterion uniquely determines the value of the compressive force, at which further operation of the structure is impossible.

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## ГЕОМЕТРИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДИАГРАММ ДЕФОРМИРОВАНИЯ И КРИТЕРИЯ ПРОЧНОСТИ КОМПОЗИТНЫХ СТАЛЕБЕТОННЫХ СТЕРЖНЕЙ



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*Ключевые слова:* сталебетонные стержни, напряженно-деформированное состояние, диаграммы деформирования, критерий прочности, разрушение.

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*В данной статье представлены математическая модель и геометрический критерий определения несущей способности сталебетонных стержней. Предложена функция деформирования композитных стержней, позволяющая определять значения критических нагрузок. Полученные результаты могут быть использованы в пакетах прикладных программ для автоматизированного проектирования.*

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