

УДК 624.072.22

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## **VERIFICATION AND REFINEMENT OF THE METHODOLOGY FOR CALCULATING THE STIFFNESS OF JOINTS OF CRANE SECONDARY TRUSS IN ITS PLANE**

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*Key words:* crane secondary truss, elastic supports, linear pliability, stiffness, bottom chord, stiffness of elastic supports.

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*In this paper, the results of verification of the proposed methodology are presented, and refined formulas for calculating the compliance of the crane-way chord joints and the stiffness of elastic supports for crane secondary trusses with a different number of panels are presented.*

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### **Introduction**

Crane secondary trusses (CST) are structures that combine the functions of crane systems (systems that support crane rails and allow overhead cranes to move along them) and the functions of main truss structure that supports secondary trusses and roof structures. Longitudinal CST are used when overhead cranes are moving along the building, they are widespread in industrial buildings of metallurgical plants. Spans of such structures can reach 48 meters, crane load capacity is over than 400 tons, crane operation modes at metallurgical plants are 7K, 8K. CST elements are made of dimensional welded I-beams and closed box sections with significant stiffness both in and out of the CST's plane.

The spatial work of the CST is ensured primarily by a strong crane-way chord of closed cross-section, which can equally work on bending in two planes and resist twisting deformations. The chord is made of thin-walled box section, often asymmetrical, with cantilevers, diaphragms, stiffeners and other reinforcing elements. The height and width of the section may exceed 3 meters [1]. Analytical calculation of the chord, on which the crane moves, is complicated by the fact that local concentrated crane loads applied to the chord with eccentricity cause its constrained torsion. Thin-walled beams of closed profile in torsion are in a complex stress-strain state (SSS), additional normal stresses and deformations occur in them [2]. The classical theory calculation of thin-walled beams with a closed profile was developed by A. A. Umansky [3]. Studies of the SSS of thin-walled beams with a closed profile and various methods of analysis are proposed in [4–7].

In accordance with the recommendations for the calculation of CST that are given in [8]:

– normal forces of the elements of the CST are determined by the design scheme, which is a truss with hinged joints and with the struts centered on the axis of the crane-way chord;

– bending moments in the crane-way chord consist of three components: moments in an unsplit beam on rigid supports; moments from the deflection of the truss due to the unsplit nature of the crane-way chord; moments from the eccentric connection of the lattice elements to the crane-way chord [9];

– the stiffness of joint connections of the crane-way chord with the CST's web members is not considered in the calculation [10].

The paper [11] proposes a refined design scheme of the crane-way chord, that takes into account the elastic pliability of the web in the plane of the truss, presents a method for determining the stiffness of the elastic supports emitting the work of the web and the results of an analytical method for selecting the cross-section and determining the stiffness of the elastic supports of the chord in the plane of the truss. This paper presents the results of verification of the methodology [11] on the CSTs with different geometrical parameters. The refined formulas for calculating the ductility of crane-way chord joints and the stiffness of the elastic supports are presented for CSTs with different number of panels.

### Research Methods

The objects of validation of the methodology [11]:

1. CST No. 1 with a span of 36 m, located in the foundry of a metallurgical plant (Fig. 1);
2. CST No. 2 with a span of 36 m, made by TSNIISK for the Cherepovets metallurgical plant (Fig. 2);
3. CST No. 3 with a span of 48 m, designed for the same plant (Fig. 3);
4. CST No. 4–9-meter CST designed by TSNIISK for the purpose of experimentation (Fig. 4).

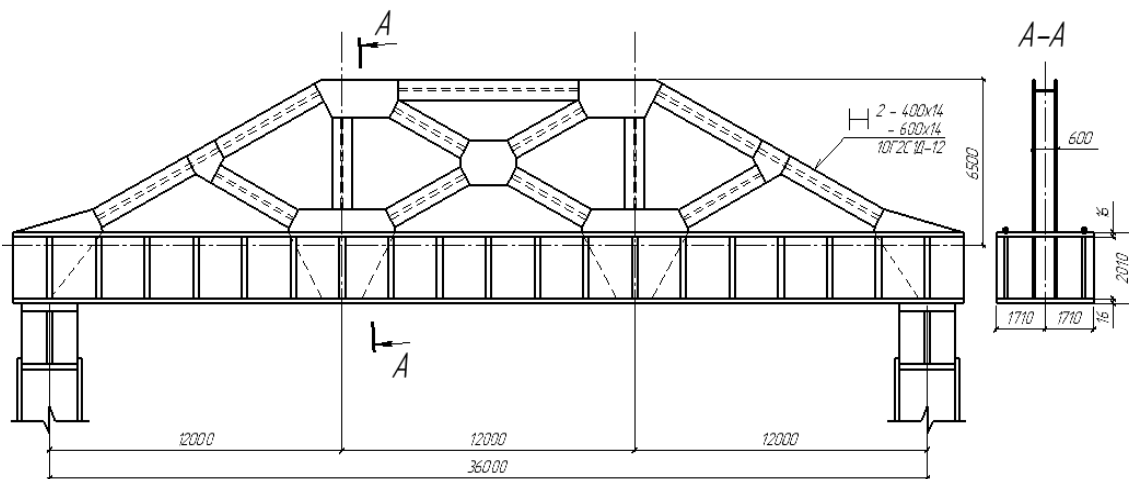


Fig. 1. CST No. 1

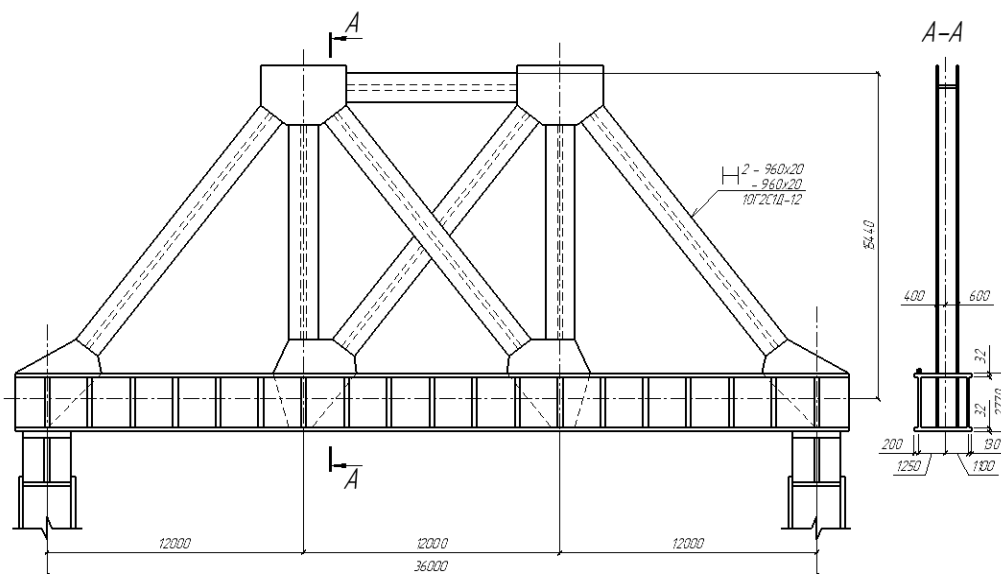


Fig. 2. CST No. 2

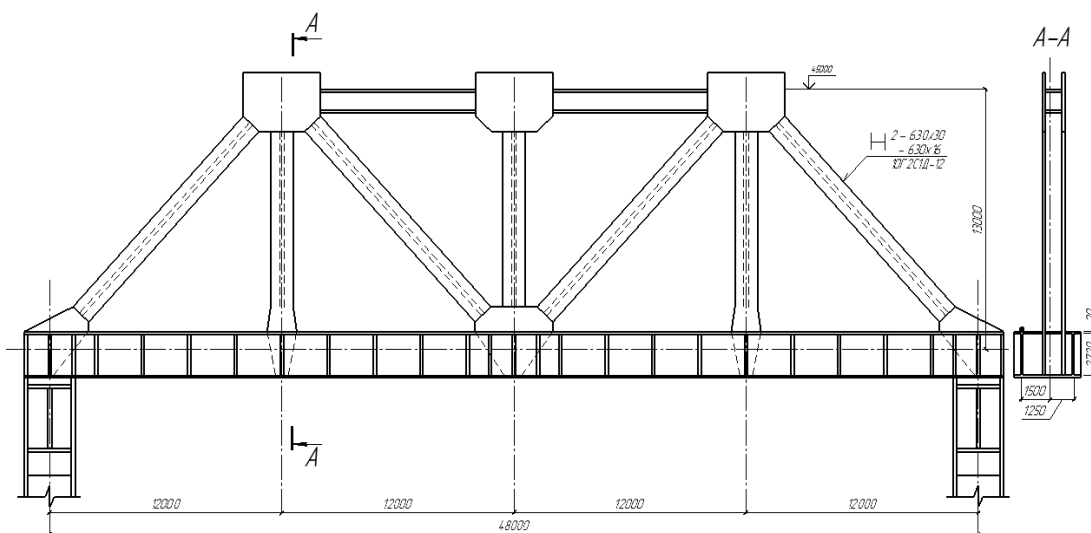


Fig. 3. CST No. 3

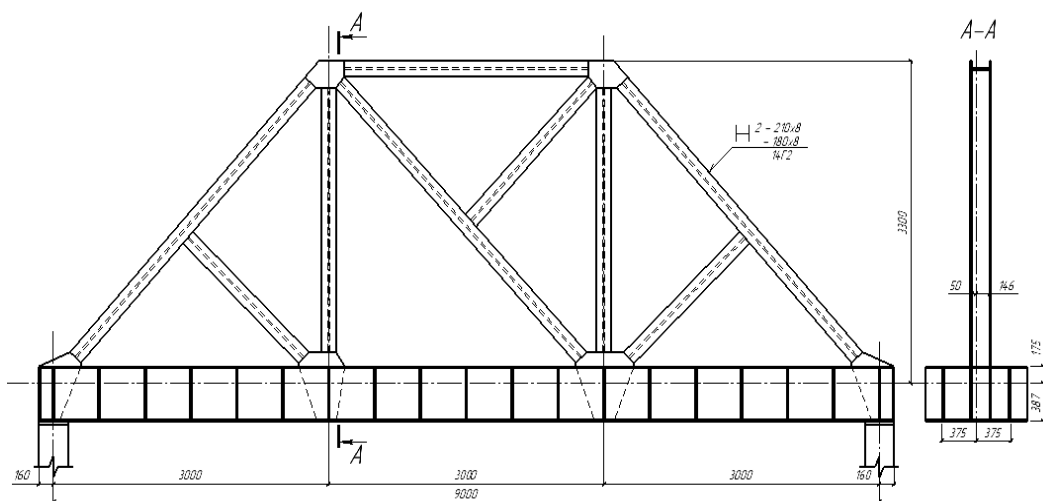


Fig. 4. CST No. 4

Analytical and numerical methods were used for each CST to determine the linear compliance of the joints of the web members adjoining the crane-way chord in the plane of the CST and the stiffness of the elastic supports that emit the work of these joints. The numerical calculation was performed in SCAD for a flat design scheme (DC) with rigid joints and considering the eccentricity of the web members fixing by rigid inserts [12]. The analytical method was calculated using the formulas [11]:

$$\delta_{CST} = \frac{2b \sum_1^m [M_{iBC}]^2}{3EI_{yBC}} + \frac{\sum_1^m l_{iw} [N_{iw}]^2}{EA_w}, \quad (1)$$

$$C_p = \frac{EA_w}{\sum_1^m l_{iw} [N_{iw}]^2} + \frac{3EI_{yBC}}{b} \left( \frac{1}{2 \sum_1^m [M_{iBC}]^2} - \frac{n}{(n-1)^2 b^2} \right) \left[ \frac{\text{кН}}{\text{м}} \right], \quad (2)$$

$n$  – number of CST's panels;

$b$  – bottom chord panel length;

$M_{iBC}$  – bending moments in the  $i$ -th node of the bottom chord of CST;

$N_{iw}$  – longitudinal forces in  $i$ -th web members and top chord;

$l_{iw}$  – length of the  $i$ -th web member or top chord;

$I_{yBC}$  – moment of inertia about an  $y$  axis at the bottom (crane-way) chord;

$A_w$  – area of web members and top chord.

For CST No. 3, the calculation of joint's compliance and stiffness of elastic supports was performed for joints No. 1 and No. 2 (Fig. 5a).

Because of obtaining a high error in the results of the calculation of the compliance of the joint No. 2 an additional study of its components was performed in order to refine the calculation method [11]. The deflections and internal forces of CST No. 3 were determined by analytical method using the internally statically indeterminate hinge-rod model with a non-split bottom chord (Fig. 5b). The deflections  $f_{\Pi\Pi\Phi}$  are calculated using the Moore-Maxwell formula:

$$f_{CST} = \sum_1^m \int_0^l \frac{[\overline{M_{BC}^0}]^2}{EI_{yBC}} ds + \sum_1^m \int_0^l \frac{[\overline{N_i^0}]^2}{EA} ds [m], \quad (3)$$

$\overline{M_{BC}^0}$  и  $\overline{N_i^0}$  – bending moment in the bottom chord elements and longitudinal forces in the CST rods caused by the action of a unit force  $P = 1$ , applied in the direction of the desired displacement;  $l$  – length of the truss rod, the summation is performed for all rods.

The comparison of internal forces in CST No. 3 obtained by analytical and numerical methods of calculation is carried out, the accuracy of determination of displacement components is analyzed.

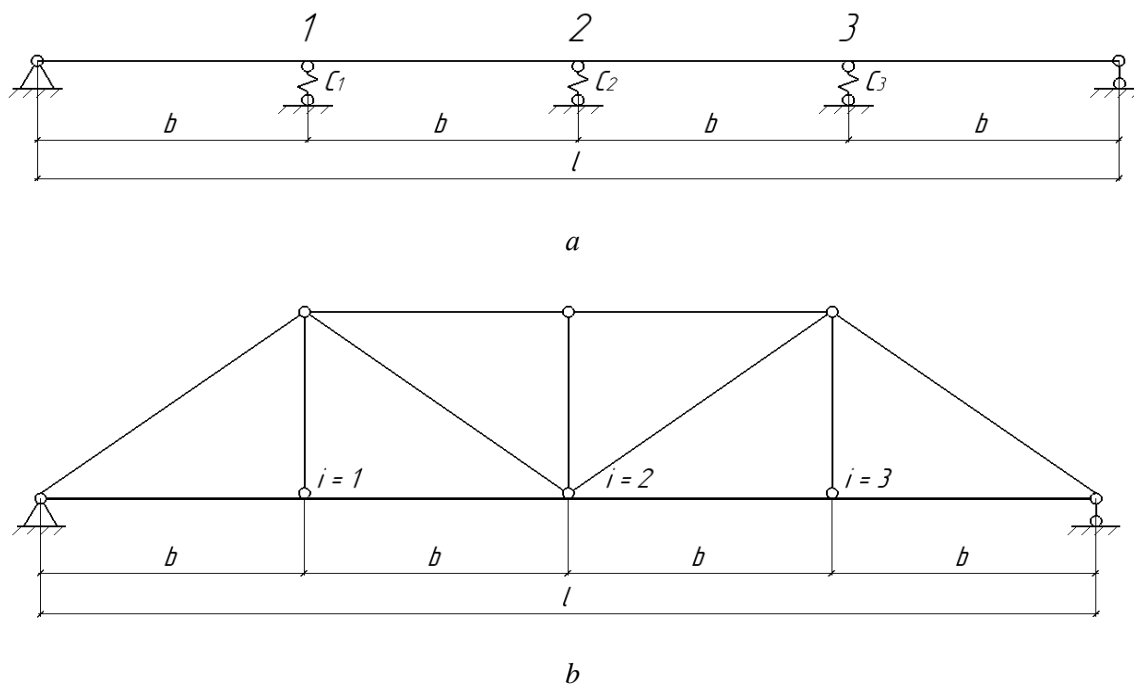


Fig. 5. DS for CST No. 3 for analytical calculation: *a* – equivalent to the crane-way chord on elastic supports; *b* – with non-split bottom chord

### Results

Comparison of the results of the methodology [11] and numerical determination of joint compliance and stiffness of elastic supports for the researched CSTs are shown in Table 1 and Figs. 6–8.

Table 1

### Comparison of the results of the method [11] and numerical determination of the joint compliance and stiffness of the elastic supports of the bottom chord in the plane of the CST

No CST	Determination of the CST, mm		Error in determining displacements, %	Stiffness of elastic supports, kN/mm		Error in determining displacements, %
	method [11]	numerical calculation		method [11]	numerical calculation	
1	3,273	3,22	1,63	272	277	-1,86
2	2,003	1,98	1,14	411	417	-1,41
3 node 1	2,683	2,6	3,08	308	320	-3,85
3 node 2	4,295	3,755	12,57	197	230	-17,03
4	0,765	0,76	0,67	676	685	-1,31

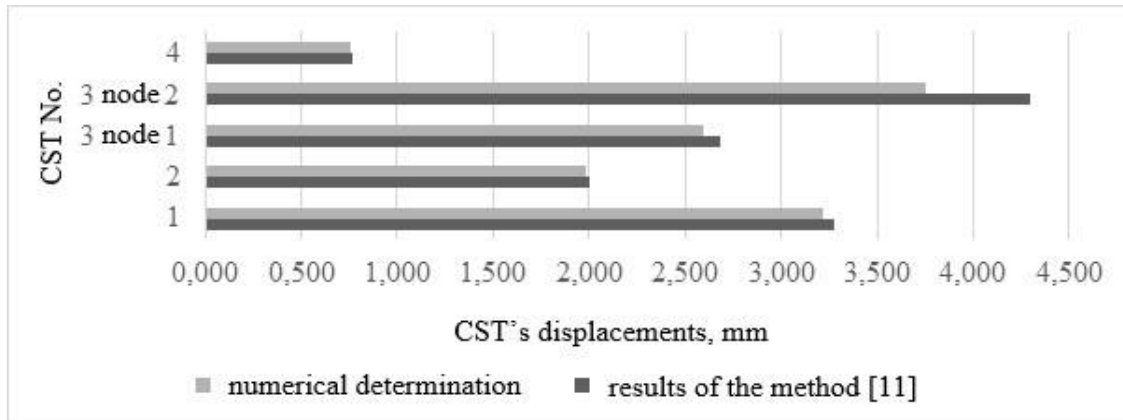


Fig. 6. CST's displacements

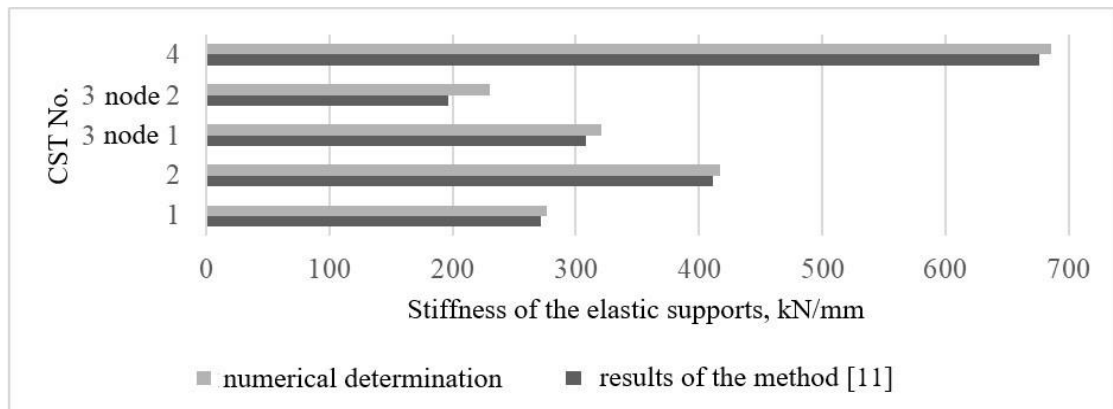


Fig. 7. Stiffness of the elastic supports of the bottom chord of the CST

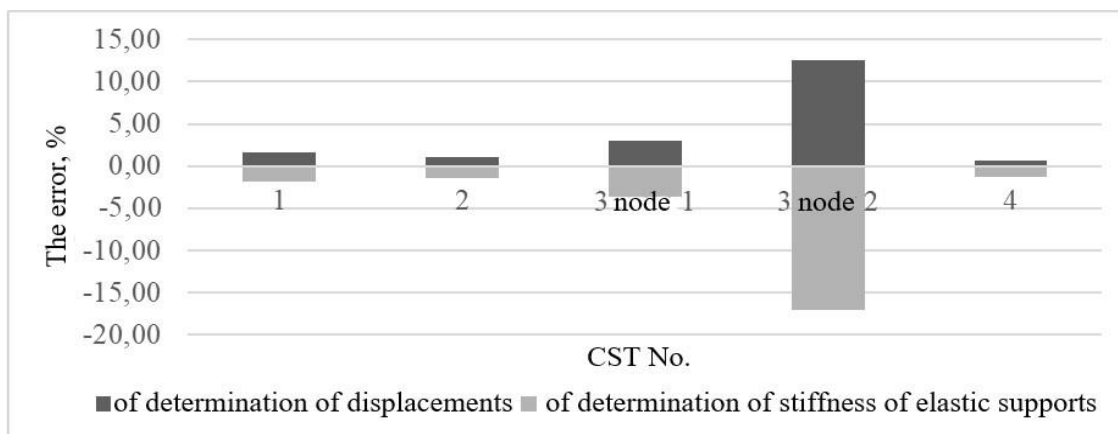


Fig. 8. Accuracy of determination of displacements and stiffness of elastic supports of the bottom chord of the CST



The error of the methodology [11] for determining the compliance of the nodes and the stiffness of the elastic supports of CSTs No. 1, No. 2 and No. 4 does not exceed 2 %. For CST No. 3 the error in determining the stiffness of elastic supports for nodes No. 1 and No. 3 is 4 %, for node No. 2-17 %. It can be predicted that the deviation of the method results from the numerical study will increase with the increase in the number of BCP panels.

The comparison of the longitudinal forces in the web and top chord elements  $N_{iw}$  and bending moments in the bottom chord elements  $M_{iBC}$ , obtained by numerical and analytical methods, were carried out in order to refine the calculation methodology. The deflection and internal forces, which were obtained by the analytical method, were determined using an internally statically indeterminate hinge-rod model with an uncut bottom chord (Fig. 5b).

The results of the determination of the compliance of the bottom belt of CST No. 3 are given in Table 2 and in Fig. 9.

Table 2

**Comparison of the results of analytical and numerical methods for determining the compliance of the bottom chord nodes in the CST plane**

Node No, $i$	Compliance from bending of the lower chord, mm		CST's compliance, mm		
	formula (4)	formula (5)	formula (6)	formula (7)	numerically
1	0,700191778	0,690	2,693	2,683	2,6
2	0,641256143	1,153	3,783	4,295	3,755

The accuracy of displacement component's determination is analyzed. According to the analysis, the error in determining the compliance of the nodes and the stiffness of the elastic supports in the plane of the CST obtains due to the simplification of the determination of the displacement from the bending of the bottom chord.

$$\delta_{CST \text{ from } M_{BC}} = \sum_1^m \int_0^l \frac{[\overline{M_{BC}^0}]^2}{EI_{yBC}} ds, \quad (4)$$

in the form of

$$\delta_{CST \text{ from } M_{BC}} = \frac{2b \sum_1^m [M_{iBC}]^2}{3EI_{yBC}}. \quad (5)$$

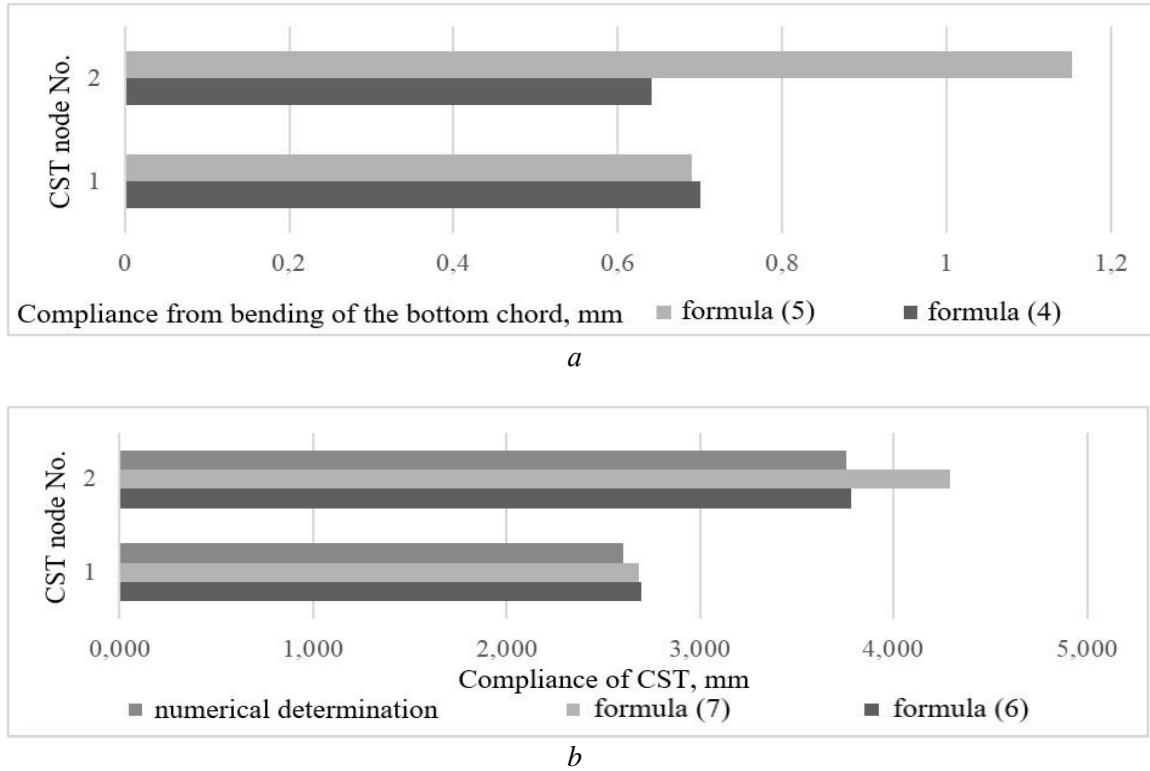


Fig. 9. Compliance of CST No. 3: *a* – from bending of the bottom chord; *b* – total

After the calculation using formula (6) for CST No. 3, the error in determining the total compliance relative to the numerical calculation method for node No. 2 is 0.7 %, for node No. 1–3.5 %.

The following formula should be used to calculate the compliance of the nodes of a crane-way chord consisting of 4 or more spans:

$$\delta_{CST} = \sum_1^m \int_0^l \frac{[\overline{M_{BC}^0}]^2}{EI_{yBC}} ds + \frac{\sum_1^m l_{iw} [N_{iw}]^2}{EA_w}, \quad (6)$$

*i* – number of the web's joint with the bottom chord (excluding the support one) – fig. 5.

For the nodes nearest to the supports ( $i = 1; i = n - 1$ ) of the three- and two-panel CST the formula (6) can be simplified:

$$\delta_{CST} = \frac{2b \sum_1^m [M_{iBC}]^2}{3EI_{yBC}} + \frac{\sum_1^m l_{iw} [N_{iw}]^2}{EA_w}. \quad (7)$$

General view of the formula for determining the stiffness of elastic supports emitting the work of the grid attachment points in the CST plane:

$$C_i = \frac{EA_w}{\sum_1^m l_{iw} [N_{iw}]^2} + EI_{yBC} \left( \frac{1}{\int_0^l [\overline{M_{BC}^0}]^2} - \frac{3n}{(n-i)^2 i^2 b^3} \right) \left[ \frac{kN}{mm} \right]. \quad (8)$$





For the joints of the bottom chord with the web ( $i = 1; i = n - 1$ ) closest to the supports of a three- and two-panel CST, formula (8) is simplified to the form:

$$C_i = \frac{EA_w}{\sum_1^m l_{i_w} [N_{i_w}]^2} + \frac{3EI_{y_{BC}}}{b} \left( \frac{1}{2 \sum_1^m [M_{i_{BC}}]^2} - \frac{n}{(n-1)^2 b^2} \right) \left[ \frac{kN}{mm} \right]. \quad (9)$$

### Conclusions:

1. Verification of the methodology for determining the compliance of the crane-way chord nodes and the stiffness of the elastic supports emitting the work of these nodes in the plane of the CST for different design solutions of the CST has been performed.

2. Clarification of the formula for calculating the stiffness of elastic supports for CST with different number of panels is carried out.

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### **ПРОВЕРКА И УТОЧНЕНИЕ МЕТОДИКИ РАСЧЕТА ЖЕСТКОСТИ УЗЛОВ ПОДКРАНОВО-ПОДСТРОПИЛЬНОЙ ФЕРМЫ В ЕЕ ПЛОСКОСТИ**

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*Ключевые слова:* подкраново-подстропильная ферма, упругие опоры, линейная податливость, жесткость, ездовой пояс, жесткость упругих опор.

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*Приведены результаты проверки предложенной методики, представлены уточненные формулы расчета податливости узлов ездового пояса и жесткости упругих опор для подкраново-подстропильных ферм с различным количеством панелей.*

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Получено: 29.11.2023 г.